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# Quality, Trade and the Moving Windows: Competitiveness and the Globalization Process

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## **Abstract**

This paper analyses the globalization process by reference to a model in which firms and countries differ both in productivity and quality. This model is representative of a class of models popular within the Industrial Organization literature, that have proved successful in explaining cross-industry differences in market structure. The model, and its implications, differ fundamentally from those of monopolistic competition (Dixit-Stiglitz-Krugman) type that are now standard in the Trade literature. One implication of the present model is that there is a lower bound to quality below which firms cannot sell, however low the (local) wage rate they face. Another implication is that the range ('window') of quality levels between the current maximum level and this lower bound shifts upwards when trade is liberalized. A third set of implications throw new

light on the ‘competitiveness’ debates of the 1990s.

## 1 Introduction

This paper explores a model of trade in which firms differ in respect of both productivity and quality. The model is standard within the Industrial Organization literature but differs in important ways from models such as the CES type models of Dixit-Stiglitz (1977), Krugman (1978), and Fujita, Krugman and Venables (1999) that are standard in the trade literature.

The model of product market competition on which the present analysis rests is one of a broad class of models that are used in the modern literature on market structure<sup>1</sup>. This class of models is characterized by two key properties: consumers choose which firms product to buy on the basis of the price-quality combinations on offer, and

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<sup>1</sup>The CES models and related models arise as special limiting cases of the general class of models, within which certain results central to the present paper vanish. The main argument in favour of the type of model used in the present paper is that it leads to empirically successful predictions regarding the levels of market concentration found across different industries; while appealing to a special ‘limiting case’ models such as DSK does not. Indeed the latter kind of model cannot be reconciled with the fact that many industries remain concentrated even in very large (global) markets. (For a review of the literature, see for example Sutton (2007)). This contrasts with a CES setting, in which each customer does not choose a single preferred product, but spreads his or her purchases over all products, buying more (or less) of the product depending on its price-quality combination, so that as the number of product varieties becomes arbitrarily large, the market share of each variety, including the highest quality variety, shrinks to zero.

- (i) any firm can choose to improve its level of productivity and/or quality by spending fixed and sunk costs outlays ('R&D')<sup>2</sup>.
- (ii) If one firm's product has a quality level superior to, (and the same unit cost of production as) its rivals, then it will retain some (strictly positive, minimal) level of market share, even in the limit where the number of low quality rivals becomes arbitrarily large. In other words, 'high quality goods cannot be squeezed out by low-quality goods'. This can be shown to be equivalent to saying that, even as a low quality rival's prices fall to the level of unit variable cost, customers will still be willing to pay a strictly positive price premium for the high quality good.

It is well known from the I.O. literature that, within such a setting, there will be some lower bound to market concentration, no matter how large the market becomes. This happens because increases in market size induce an increase in fixed and sunk outlays by currently active firms, rather than inducing the entry of new firms (Sutton, (1991, 1998)).

This equilibrium level of concentration is, in general, consistent with the existence of an arbitrarily large number of firms, so long as some given fraction of consumers ignore quality, and buy on the basis of price alone. In the present

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<sup>2</sup>If quality improvements are associated, not fixed and sunk outlays, but rather with a rise in variable costs (labour and raw material inputs), then 'quality competition' is precisely analogous to competition in product variety of the Hotelling kind (i.e. to 'horizontal' product differentiation). This kind of setup is captured, for example, in Rosen (1978); for a discussion, see Sutton (1991), Chapter 3.

paper, we simplify matters by letting all consumers have identical tastes; under this assumption, the familiar propositions on the level of concentration emerge as statements about the equilibrium number of firms in the market.

The first step in the present analysis lies in examining, in a single-economy, partial equilibrium setting, how the effect of differences in quality and productivity affect firms' survival. We define a 'capability' index, which combines information about a firm's productivity and quality levels, and we show that, at equilibrium, there will be a range, or 'window', of capability, running from the highest level attained by any firm, down to a threshold level determined by the process of competition between surviving (i.e. active) firms. Firms with capabilities below the threshold will have zero output at equilibrium (i.e. be inactive) (Section 2).

The main body of the paper is concerned with analysing, in a multi-country general equilibrium setting, the effect of opening up trade between hitherto separated economies, each of which has many industries of this kind. We distinguish three phases, associated with distinct competitive mechanisms, as follows:

1. The 'impact' phase: Here, we take all firms' levels of productivity and quality as fixed. The effect of liberalization is to induce a selection effect across firms and countries, under which low capability firms suffer a shakeout. The key question of interest relates to the way in which the loss of some firms or industries in a country lowers its

wage rate relative to other countries, and the way in which this wage reduction can or can not compensate for (poor) levels of productivity and quality in other industries (Section 3).

The central message relates to a contrast between productivity and quality. Differences in productivity can be fully offset by wage differences, in the manner of the conventional literature. Differences in quality can not: the cost of imported raw materials<sup>3</sup> sets a floor to unit cost, and so price, independent to the local wage rate; and it follows from this that there is, at equilibrium, a floor to the firm's quality level below which it cannot survive even in a very low-wage economy (The 'quality window')<sup>4</sup>.

This phenomenon runs counter to one of the central arguments laid out by Krugman (1994,1999) in the course of the 'competitiveness debate' in the 1990s; in the models considered here, unlike the models Krugman had in mind in the course of that debate, it is *not* the case that *absolute* levels of firms' capability translate into countries real wage and welfare levels. Rather, *relative* quality levels matter, not

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<sup>3</sup>More generally, materials or component inputs that are internationally traded.

<sup>4</sup>It is worth noting that a (loose) analogy exists between the way competition in quality works in present model, and in the O-Ring model of Kramer (1999). The O-Ring model rests on a special assumption as to how the quality of a component inputs determine the quality of the final product. Here, we make no assumptions of this kind; all that is needed is the notion that a consumer is willing to pay, for any given quality gap over rival products, some corresponding positive price premium, independently of the number of low quality alternatives offered.

only at the (uncontroversial) level of the firm, but at the level of countries too.

An implication of this is that the initial ‘impact’ phase of the globalization process can be welfare-reducing for intermediate-capability countries.

2. The transfer phase: The wage and capability differentials across countries in phase 1 creates a strong profit incentive for firms to transfer of high-level capabilities to low-wage countries. In Section 4, we examine the impact of such transfers, which can occur through various routes. The central argument of this paper is that it is this second phase of the process which carries the main benefits of globalization to low wage economies. The determinants of the speed and effectiveness of this process, it is argued, are only partially understood. Such research as is available suggests (i) that the speed and effectiveness of the process varies widely across industries, and that (ii) one systematic influence that plays a major and general role relates to the vertical transfer of capabilities within the host (low-wage) economy through the supply chain of multinational producers.
3. The escalation phase: Overlapping in time with phase 2, this third phase involves the re-adjustment of firms to the global environment. Their pre-globalization levels of investment in building their productivity and quality levels were predicated on access to a limited number of markets. Now that they have access to wider markets, the optimal

level of fixed and sunk outlays aimed at enhancing productivity and quality rises. Just as the bound to the number of firms in the pre-globalization setup reflected the fact that firms' equilibrium outlays on productivity and quality enhancement raised the costs of competing in the domestic market, so too will renewed escalation of fixed and sunk outlays induce a new shakeout of firms in the globalized market. (Section 6)<sup>5</sup>

Finally, one caveat is in order: the gains from trade liberalization derive from several distinct channels, which include: (i) benefits of specialization that reflects differences in factor endowments, i.e., classical 'comparative advantages', and (ii) benefits of 'scale and variety; associated with 'horizontal' product differentiation, as captured by the now-standard monopolistic competition models, following Krugman (1978). In the present paper, both these channels are omitted, the first by eliminating differences in factor endowments, and the second by using a 'pure vertical product differentiation' setting. The reason for adopting these two simplifications is to focus attention on what is novel here, at the cost of understating the gains from trade.

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<sup>5</sup>We may illustrate the mechanisms in phases 1 and 3 above in a single country by examining the quality (or capability) window in a setting where a new (high-level) entrant arrives in an industry: the arrival causes equilibrium prices to fall, thus raising the lower bound required for survival ('Shakout'; mechanism 1). The now narrower (capability or quality) gap between active firms raises the marginal returns from investment in capability building; so that some (or all) firms move upwards, thus (possibly) raising the highest level of capability attained (the 'moving window'). Essentially, the mechanism of phase 1 relates to the upward movement at the bottom of the window, while the mechanism in phase 3 is associated with the upward movement of the top.

## 2 Product Market Equilibrium

To ease exposition, we first describe product market equilibrium in a one-country, partial equilibrium setting. This allows us to introduce some basic features of the model with a minimum of notation; these features will carry over immediately to the multi-country, general equilibrium version of the model in the next section.

We consider a single industry in which  $n$  firms each offer a single product. Firms, and their respective products, are indexed by  $i$ . Firm  $i$  is characterised by a ‘productivity’ parameter  $c_i$  and a ‘quality’ parameter  $u_i$ . We refer to the pair  $(u_i, c_i)$  as firm  $i$ ’s ‘capability’. All firms face a common wage rate  $w$  and a price per unit of raw materials (or intermediate inputs)  $p_0$ . Firm  $i$  has unit variable cost (i.e. a constant marginal cost) equal to the sum of a wage cost  $wc_i$  and a cost of materials (or intermediate inputs)  $p_0\mu$ , where  $\mu$  is the quantity of materials input per unit of final output). Total consumer expenditure, denoted  $S$ , is wholly devoted to the consumption of this good. All consumers have the same utility function  $\bar{U} = ux$ , where  $u$  denotes quality and  $x$  is the quantity consumed. It follows that the consumer chooses (only) the good(s) with the lowest price-quality ratio. We seek a Nash equilibrium in quantities (Cournot equilibrium). This is computed as follows: since all goods commanding positive sales must have the same (equal lowest) price-quality ratio, we may write



$$\frac{p_i}{u_i} = \lambda, \quad \text{all } i$$

whence if  $x_i$  denotes the output of firm  $i$ , we have

$$\sum p_j x_j = \lambda \sum u_j x_j = S$$

whence  $\lambda = S / \sum u_j x_j$  and  $p_i = u_i S / \sum u_j x_j$ .

Firm  $i$  sets  $x_i$  to maximize

$$\begin{aligned} \pi_i &= p_i x_i - (w c_i + p_0 \mu) x_i \\ &= u_i S / \sum u_j x_j - (w c_i + p_0 \mu) x_i \end{aligned}$$

taking  $x_i, \dots, x_{i-1}, x_{i+1}, \dots, x_n$  as given.

A routine calculation (following Sutton (1998), pp. xxx-xxx) yields the Nash equilibrium solution for the firms' outputs and prices, given their capabilities. It is convenient to adopt the shorthand notation  $k_j$  to represent the ("effective cost") indicator,  $(w c_j + \mu p_0) / u_j$ , and to express the solution in terms of quality adjusted prices and outputs as follows:

$$\frac{p_i}{u_i} = \frac{1}{n-1} \sum_{j=1}^n k_j \tag{1}$$

$$u_i x_i = S \frac{n-1}{k_i} \left\{ 1 - (n-1) \frac{k_i}{\sum_{j=1}^n k_j} \right\} \quad (2)$$

Firm  $i$ 's equilibrium profit equals

$$\pi_i = p_i x_i - (w c_i + p_0 \mu) x_i = \left\{ 1 - (n-1) \frac{k_i}{\sum_{j=1}^n k_j} \right\}^2 \cdot S \quad (3)$$

It follows immediately from inspection of (2) that there is a critical level of  $(u_i, c_i)$  which a firm must attain in order to command positive sales at equilibrium. In characterizing this condition, it is convenient to focus on a setting in which there are  $n+1$  firms; of these  $n$  are 'active' in the sense of producing positive output at equilibrium, while the  $(n+1)$ th firm is on the margin of viability, i.e. the equilibrium value of  $x_{n+1}$  is zero. Re-writing equation 2 for  $(n+1)$  firms and setting the r.h.s. to zero, we have on re-arranging:

$$k_{n+1} = \frac{n}{n-1} \cdot \frac{\sum_{j=1}^n k_j}{n} \quad (4)$$

where the final ratio on the r.h.s. is the average value of the 'effective cost' indicator for the  $n$  active firms. Thus the condition for the viability of the marginal entrant to the industry is that its effective cost indicator should not exceed that of the industry average value by more than the factor  $n/(n-1)$ .

It will be of interest in what follows to ask: to what extent can a fall in  $w$  raise a firm with a given capability to the threshold of viability defined by (4). Noting that  $k_{n+1} = (wc_{n+1} + p_o\mu)/u_{n+1}$ , it follows that the schedule in  $(1/c, u)$  space corresponding to equation (4) takes the form of a right-angled hyperbola

$$u_{n+1} = \frac{1}{a} \cdot \left( \frac{w}{1/c_j} + \mu p_0 \right)$$

where  $a = \left( \sum_{j=1}^n k_j \right) / n$  denotes the industry average value of the effective cost ratio. The horizontal asymptote  $\mu p_0 / a$  denotes a level of quality below which the firm is non-viable, however low the wage rate it faces: in the limit  $w \rightarrow 0$  the schedule collapses to the form indicated in Figure 1.

Once we turn to a multi-country setting, it will be of interest to ask: to what extent can low domestic wages render low-capability firms viable? What Figure 1 suggests, and what will emerge as a basic result in the general equilibrium setting of Section 3, is that low wages can always fully compensate for any deficiency in productivity (the threshold collapses to  $1/c = 0$  as  $w \rightarrow 0$ ) but not for deficiencies in quality (the threshold remains strictly positive in the limit). The intuition is as follows: poor (labour) productivity is reflected in high unit labour cost, but labour cost falls to zero with  $w$  whatever the value of  $c$ . On the other hand, poor quality can be offset by low prices, but so long as the firm must purchase some inputs from outside, its unit cost is bounded away

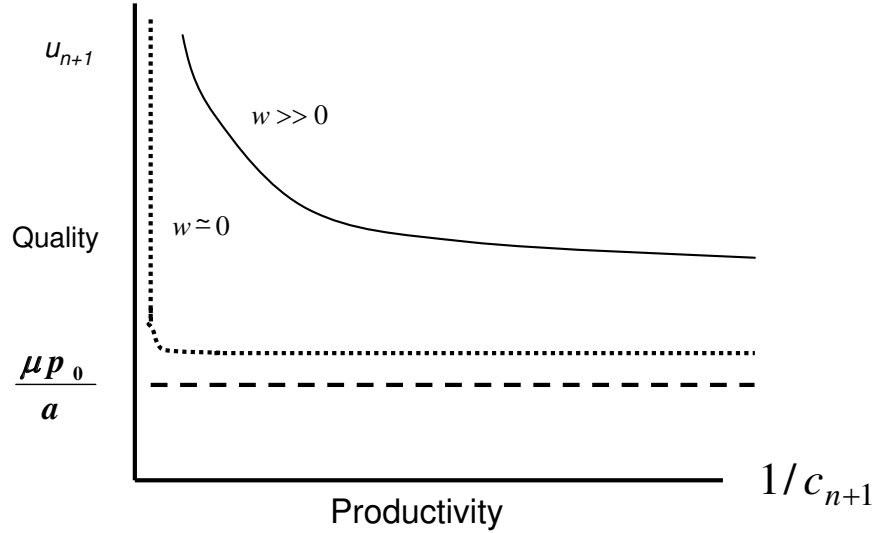


Figure 1: The threshold of viability in  $(1/c, u)$  space. The dotted line indicates the limiting form of the schedule as  $w \rightarrow 0$ .

from zero even when  $w = 0$ ; and if the quality level falls so low that it cannot support the price of  $p = \mu p_0$  that covers its unit material cost, then the product is non-viable. We may sum this up as:

**Proposition 1** (*The short run threshold*). *For any set of  $n$  firms with capabilities  $(1/c_i, u_i)$  there is a region in  $(1/c, u)$  space below which no firm (or potential entrant) can achieve viability. In the limit  $w \rightarrow 0$ , the schedule collapses to zero in  $1/c$ , but is bounded away from zero in  $u$ .*

Proposition 1 is central to the argument that follows; the main task of the next section is to extend this proposition to a setting of general equilibrium in a multi-country model. Before continuing, however, we digress briefly in order

to complete our characterization of product market equilibrium, by posing a question that will play an important role in our discussion of the globalization process (Section 4): what determines the number of firms that will enter the industry in the long run? In particular, how does this number vary with the size of the market  $S$ ?

So far, we have taken firms' capabilities as given; this is appropriate in a 'short run' setting. But in the long run, firms may invest in fixed and sunk outlays, whether through product or process innovation, to improve their levels of  $u$  and  $1/c$ . This leads to the question: suppose all firms face some common schedule  $F(1/c, u)$ , then (a) how many firms will enter the market at equilibrium, and (b) what can we say about their levels of  $(1/c, u)$ ?

We proceed, following Sutton (1991, 1998), by assuming that the firm can attain a capability  $u, 1/c$  by incurring a fixed and sunk cost

$$F(u, 1/c) = u^\beta + c^{-\gamma}$$

where the elasticity parameters  $\beta$  and  $\gamma$  measure the effectiveness of product and process innovation respectively. We model this process as a 3-stage game. In stage 1, each of a number of potential entrants chooses either not to enter; or else it chooses to enter, paying a setup cost corresponding to the fixed and sunk outlay required to establish a plant of some minimal level of capability  $(u_0, 1/c_0)$ . In stage 2, having observed the number of firms that have entered, each firm chooses its level of  $u$  ( $\geq u_0$ ) and  $c$  ( $\leq c_0$ ), and it incurs the incremental cost  $F(u, 1/c) - F(u_0, 1/c_0)$  accordingly. In stage 3, capabilities are given, and

firms compete à la Cournot as described above. Equilibrium is characterised as a (sub-game) perfect equilibrium of this 3-stage game.

An increase in market size might in principle lead either to a rise in the number of firms, or to a rise in the fixed outlays incurred by each firm, or both. A well known result is that the latter effect dominates: an indefinite rise in  $S$  will not lead to a fragmented market structure. Rather, the level of concentration (measured in the present ‘symmetric’ setting by the reciprocal of the number of firms) will remain bounded as the size of the market increases. This result is stated as:

**Proposition 2** *The equilibrium value of  $n$  lies between two limiting values,  $\underline{n}$  and  $\bar{n}$ , for all  $S$ . For  $S \rightarrow \infty$ , it approaches a value  $\underline{n}$  defined implicitly by*

$$\underline{n} + \frac{1}{\underline{n}} - 2 = \frac{1}{2} \frac{1}{\frac{1}{\beta} + \frac{1}{\gamma}}.$$

*In the limit  $S \rightarrow \infty$  it converges to an asymptotic value  $\bar{n}$ , defined implicitly by the equation*

$$\bar{n} + \frac{1}{\bar{n}} - 2 = \frac{1}{2}\beta.$$

Proof: Appendix 1

Remark: The intuition is as follows: as  $S \rightarrow \infty$ , firm  $i$ 's spending to reduce  $c_i$  will lead to  $wc$  becoming small relative to  $\mu p_0$ , and the returns from further

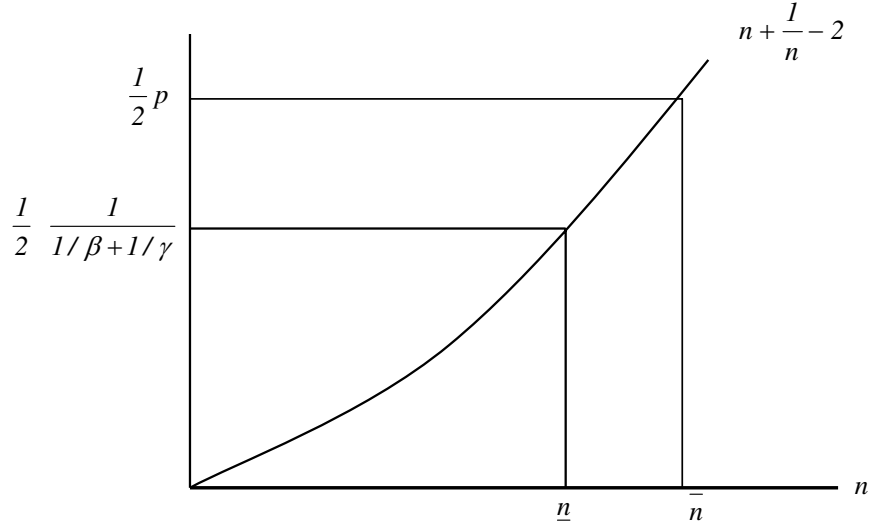


Figure 2: Market Size and Firm Numbers. The equilibrium value of  $n$  rises monotonically with market size  $S$ , increasing from an initial value  $\underline{n}$  to an asymptotic value  $\bar{n}$  as  $S \rightarrow \infty$ . The values  $\underline{n}$  and  $\bar{n}$  are illustrated

spending become increasing unattractive as unit production costs approach their minimal level  $\mu p_0$ . Increases in  $S$ , however, continue to induce spending on raising  $u$ ; and so increases in market size are accompanied, in the limit, not by a rise in firm numbers, but a rise in fixed outlays on quality improvement.

### 3 General Equilibrium in a Multi-Country Model

We consider a model in which two countries, labelled  $A$  and  $B$ , each has  $m$  ‘final goods’ industries of the kind described in the preceding section. A third country, labelled  $C$ , supplies a raw material to firms in  $A$  and  $B$  which is used in

the production of all final goods. Our focus of analysis will be on the question of how country  $B$ 's relative wage rate and level of welfare vary, as the capabilities of its firms change, holding constant the capabilities of  $A$ 's firms.

The product markets in countries  $A$  and  $B$  are identical to those considered in the one-country model above. Of the  $m$  final goods industries in each country,  $r$ , labelled  $1, \dots, r$ , are 'commodity' products, for which productivity and quality are uniform across all firms and countries. We label their quality as 1 and their unit cost of production, in terms of labour input, as  $c_1$ . We assume the number of (potential) producers each of these good to be the same in both countries, and we denote this number by  $n_1$ . Our focus of interest will lie in the case where  $n_1$  is large<sup>6</sup>.

We denote by  $u_A$  the (common) quality level attained by all firms in Country  $A$  in the production of goods  $r + 1$  to  $m$ ; and by  $c_A$  their (common) unit cost of production in labour units. The corresponding quality and productivity parameters for country  $B$  are denoted  $v$  and  $c_B$ . We denote by  $n$  the number of (potential<sup>7</sup>) producers of each of these goods, in each country.

The production of a unit of any of these goods requires  $\mu$  units of an (internationally traded) raw material for each unit of labour input. This raw

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<sup>6</sup>In terms of the model introduced in the preceding section, this is motivated as follows: when fixed and sunk outlays are ineffective in improving productivity and quality ( $\beta \rightarrow \infty, \gamma \rightarrow \infty$ ), we can treat entrants as facing a constant ('exogenous') setup cost to enter the industry. As the size of the economy increases, the number of firms rises indefinitely.

<sup>7</sup>Some of these firms may be inactive at equilibrium.



material is supplied from country  $C$ . We assume that the number of producers of this intermediate good in country  $C$  is large, and each of these firms can produce one unit of raw material using one unit of labour input; at equilibrium, this will imply that the price of the raw material input equals the wage rate in country  $C$ , which we denote as  $w_C$ .

Each country comprises a population of identical individuals ('workers'), which we assume –to ease notation– to be the same in all three countries, and which we denote by  $N$ . We assume that all profit receipts accrue to a separate group of 'owners'. Each individual ('worker') is characterized by a utility function of the form

$$U = \prod_{i=1}^m (u_i x_i)^{1/m} - \frac{1}{2} \ell^2 \quad (5)$$

For the 'owners', the utility derived from consumption of goods is the same, but  $\ell$  is fixed to be zero in (5). It follows that each of the  $N$  workers supplies an amount of labour  $\ell$ , where

$$\ell = \frac{w}{m} \prod_{i=1}^m \left( \frac{u_i}{p_i} \right)^{1/m} \quad (6)$$

$w$  is the (local) wage rate, and  $u_i, p_i$  are the qualities and prices of the goods available to them. We use as our welfare indicator the utility of the individual ('worker')<sup>8</sup>, which, using (5) and (6), equals

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<sup>8</sup>Since this is a free entry model, where gross ('final stage') profits coincide

$$U = \frac{1}{2} \left(\frac{w}{m}\right)^2 \prod_{i=1}^m \left(\frac{u_i}{p_i}\right)^{2/m} \quad (7)$$

In what follows, we examine equilibrium, first under autarky, and then under free trade.

### The Autarky Case

Here, no trade is possible between countries  $A$  and  $B$ . In order to set up an appropriate comparison with the free trade case analysed below, in which workers in country  $C$  contribute to the total demand for the final goods produced in  $A$  and  $B$ , we need to specify the access of workers in country  $C$  to these products. To simplify matters, we partition country  $C$ . One half of the workers in country  $C$  can buy from firms in country  $A$  alone, while the other half can buy from firms in country  $B$  only; and there is no movement of labour or goods (arbitrage) across the partition. (The purpose of this device is to ensure that in the symmetric case where  $A$  and  $B$  have equal capabilities, the impact of country  $C$  on outcomes in  $A$  and  $B$  is analogous to its impact in the Free Trade setting considered later).

Consider equilibrium in country  $A$ . Bearing in mind that the price of goods  $1, \dots, r$  in country  $A$  is  $w_{Ac} + w_C \mu$ , where  $w_C$  denotes the wage in the first ‘region’ of  $C$  attached to the  $A$  market. Similarly the unit cost of production of goods  $r + 1, \dots, m$  is  $w_A c_A + w_C \mu$ . It follows that equilibrium prices  $p_1$  (for

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with fixed and sunk outlays (i.e. zero net profits), up to ‘integer effects’, this seems a reasonable welfare indicator.

goods  $1, \dots, r$ ) and  $p_u$  (for goods  $r + 1, \dots, m$ ) satisfy

$$\frac{p_u}{u} = \frac{n}{n-1} \frac{w_A c_A + w_C \mu}{u}; \quad p_1 = \frac{n_1}{n_1-1} (w_A c_1 + w_C \mu)$$

and the level of welfare in country A is

$$U_A = \frac{1}{2} \left( \frac{w_A}{m} \right)^2 \prod_{i=1}^m \left( \frac{u_i}{p_i} \right)^{2/m}.$$

Substituting from the price equations, and writing the expression

$$\left[ \frac{(n-1)}{n} \right]^{2(m-r)/m} \cdot \left[ \frac{(n_1-1)}{n_1} \right]^{2r/m}$$

as  $\mathbb{N}$  to ease notation, we have

$$U_A = \frac{1}{2} \left( \frac{w_A}{m} \right)^2 \mathbb{N} \left( \frac{u}{w_A c_A + w_C \mu} \right)^{2(m-r)/m} \left( \frac{1}{w_A c_1 + w_C \mu} \right)^{2r/m}$$

The relation between  $w_C$  and  $w_A$  is as follows: from the form of the individual labour supply function (6) above, it follows that the ratio of labour supply in the first region of country C, denoted  $L_C^D$ , to labour supply in country A,  $L_A^D$  satisfies

$$\frac{L_C^S}{L_A^S} = \frac{1}{2} \frac{w_C}{w_A}.$$

Since a unit of any good produced in  $A$  requires  $\mu$  units of raw material, it follows that the ratio of labour demand in the first region of  $C$ ,  $L_C^D$ , to labour demand in  $A$ ,  $L_A^D$  satisfies

$$\frac{L_C^D}{L_A^D} = \mu$$

whence labour market equilibrium implies  $w_C = 2\mu w_A$ . We therefore have

$$U_A = \frac{\mathbb{N}}{2m^2} \left( \frac{u}{c_A + 2\mu^2} \right)^{2(m-r)/m} \left( \frac{1}{c_1 + 2\mu^2} \right)^{2r/m} \quad (8)$$

and similarly for country  $B$ .

### Free Trade

We now open up free trade between  $A$  and  $B$ , and abolish the partition between the two parts of country  $C$ . We begin with the case where all (final) goods are produced in both  $A$  and  $B$ , as happens when their capabilities are equal or close to equal. Each (final) good is now sold in a single ‘international’ market, and since each of goods 1 to  $r$  is sold by  $2n_1$  firms, and each of goods  $r + 1$  to  $m$  is sold by  $n$  firms, and all goods incur a unit materials cost of  $w_C\mu$ , it follows that the prices of goods  $r + 1$  to  $m$  satisfy

$$\frac{p_u}{u} = \frac{p_v}{v} = \frac{n}{2n-1} \left[ \frac{w_A c_A + w_C \mu}{u} + \frac{w_B c_B + w_C \mu}{v} \right] \quad (9)$$

and

$$p_1 = \frac{n_1}{2n_1 - 1} [(w_{AC_1} + w_C\mu) + (w_{BC_1} + w_C\mu)] \quad (10)$$

while the (quality-adjusted) outputs are:

$$ux_u = \frac{S}{m} \frac{2n - 1}{n} \cdot \frac{1}{\frac{w_{ACA} + w_C\mu}{u} + \frac{w_{BCB} + w_C\mu}{v}} \left\{ 1 - \frac{2n - 1}{n} \frac{w_{ACA} + w_C\mu}{u \left[ \frac{w_{ACA} + w_C\mu}{u} + \frac{w_{BCB} + w_C\mu}{v} \right]} \right\} \quad (11)$$

$$vx_v = \frac{S}{m} \frac{2n - 1}{n} \cdot \frac{1}{\frac{w_{ACA} + w_C\mu}{u} + \frac{w_{BCB} + w_C\mu}{v}} \left\{ 1 - \frac{2n - 1}{n} \frac{w_{BCB} + w_C\mu}{v \left[ \frac{w_{ACA} + w_C\mu}{u} + \frac{w_{BCB} + w_C\mu}{v} \right]} \right\} \quad (12)$$

while the output of good 1 produced in country  $A$  is

$$x_{1A} = \frac{S}{m} \frac{2n_1 - 1}{n_1} \cdot \frac{1}{(w_{AC_1} + w_C\mu) + (w_{BC_1} + w_C\mu)} \left\{ 1 - \frac{2n_1 - 1}{n_1} \frac{w_A + w_C\mu}{(w_{AC_1} + w_C\mu) + (w_{BC_1} + w_C\mu)} \right\} \quad (13)$$

and similarly for country  $B$ .

Since every unit of labour used in  $A$  and  $B$  is accompanied by  $\mu$  units of raw material, requiring  $\mu$  units of labour in country  $C$ , it follows that labour

demand in countries  $A$ ,  $B$  and  $C$  satisfies

$$L_C^D = \mu(L_A^D + L_B^D)$$

and since the labour supply equation implies that

$$L_A^S : L_B^S : L_C^S = w_A : w_B : w_C,$$

labour market equilibrium in each country implies that  $w_C = \mu(w_A + w_B)$ .

We begin by remarking on the fully symmetric case, where  $u = v$  and  $c_A = c_B$ . Note that in this case the expressions in  $\{\cdot\}$  in (11, 12) and (13) reduce to  $1/(2n)$  and  $1/(2n_1)$  respectively, and that  $w_C = 2\mu w_A = 2\mu w_B$ , whence

$$w_A c_A + w_C \mu = w_B c_B + w_C \mu = w_A (c_A + 2\mu^2)$$

and so, writing

$$\left[ \frac{(2n-1)}{n} \right]^{2(m-r)/m} \cdot \left[ \frac{(2n_1-1)}{n_1} \right]^{2r/m} \tag{14}$$

as  $\mathbb{N}'$ , we have

$$U_A = \frac{1}{2} \left( \frac{w_A}{m} \right)^2 \prod_{i=1}^m \left( \frac{u_i}{p_i} \right)^{2/m} = \frac{1}{2} \left( \frac{w_A}{m} \right)^2 \mathbb{N}' \left( \frac{u}{w_A c_A + w_C \mu} \right)^{2(m-r)/m} \left( \frac{1}{w_A + w_C \mu} \right)^{2r/m} \quad (15)$$

whence on writing  $w_C = 2\mu w_A$  becomes

$$\frac{\mathbb{N}'}{2m_2} \left( \frac{u}{c_A + \mu^2} \right)^{2(m-r)/m} \left( \frac{1}{c_A + \mu} \right)^{2r/m} \quad (16)$$

The only difference with the autarky case lies in the terms  $\mathbb{N}' > \mathbb{N}$ ; this reflects the fact that there are now twice as many producers competing in the market for each good, so that prices fall, and our welfare measure rises.

We now turn to the asymmetric case, where  $v \neq u$  and/or  $c_B \neq c_A$ . We take  $A$  as our point of reference, and examine the effect of reducing  $B$ 's capability relative to  $A$ 's. If we hold  $u$  and  $c_A$  constant, while lowering  $v$  and/or raising  $c_B$ , we pass through three successive regimes. In the first regime,  $A$  and  $B$  both produce the full range of goods, but now  $w_B < w_A$ . In the second regime,  $A$  ceases to produce the ('commodity') good 1, while  $B$  continues to produce all goods. In the third regime,  $B$  ceases to produce goods 2 to  $m$ , and it continues to be the sole producer of the ('commodity') good 1.

If  $B$ 's capability is strictly lower, i.e.  $v > u$  and/or  $c_B > c_A$ , then it is easily shown that  $w_B/w_A$  falls below unity; and in the case on which we focus, i.e. where  $n_1$  is large, the condition for viability of good 1 in country  $A$  fails; this

can be seen directly by inspection of the  $\{\cdot\}$  term in equation (13) above. In other words, the existence of any gap in capability moves us immediately into regime II.

For this reason, we begin in what follows with a detailed study of regime II; here, prices are given by equations (9) above, and the quality adjusted outputs are given by equations (11), (12); and similarly for  $B$ . For the commodity good, however, we have  $x_{1A} = 0$  and, noting that only a total of  $n_1$  producers (those in  $B$ ) now operate, it follows that

$$x_{1B} = \frac{S}{m} \frac{n_1 - 1}{n_1^2} \frac{1}{w_B c_1 + w_C \mu} \quad (17)$$

It will be helpful to begin by setting out an explicit solution for the special case where  $\mu = 0$ , which takes a relatively simple form. We introduce the symbol  $\lambda$  to represent the ratio

$$\frac{\frac{w_A c_A}{u}}{\frac{w_A c_A}{u} + \frac{w_B c_B}{v}}, \text{ whence } \frac{w_A}{w_B} = \frac{u/c_u}{v/c_v} \cdot \frac{1 - \lambda}{\lambda}. \quad (18)$$

We begin by examining labour market equilibrium. We have:

$$\begin{aligned} L_A^D &= (m - r) c_A n x_u \\ L_B^D &= (m - r) c_B n x_v + r c_1 n_1 x_{1B} \end{aligned}$$

whence

$$\frac{L_B^D}{L_A^D} = \frac{c_B}{c_A} \cdot \frac{u}{v} \cdot \frac{v x_v}{u x_u} + \frac{r}{m - r} \frac{1}{c_A} \frac{n_1}{n} \cdot u \frac{x_{1B}}{u x_u}. \quad (19)$$



Substituting for  $ux_u$ ,  $vx_v$  and  $x_{1B}$  using (11), (12) and (17), we have, following some re-arranging, that at labour market equilibrium

$$\frac{w_B}{w_A} = \frac{L_B^S}{L_A^S} = \frac{L_B^D}{L_A^D} = \frac{c_B}{c_A} \cdot \frac{u}{v} \cdot \frac{\lambda - \eta(1 - \lambda)}{(1 - \lambda) - \eta} + a \frac{\lambda}{1 - \lambda} \frac{1}{1 - \lambda} \frac{\lambda}{(1 - \lambda) - \eta\lambda} \quad (20)$$

where  $\eta$  denotes  $(n - 1)/n$  and  $a = \frac{r}{m-r} \frac{n_1-1}{n_1} \frac{1}{2n-1}$

Substituting for  $w_A/w_B$  from (18) and re-arranging, we obtain a solution for  $w_A/w_B$ , which we write as  $\omega$ :

$$\omega - \frac{1}{\omega} = \eta\left(k - \frac{1}{k}\right) - \frac{a}{k} \left(1 + \frac{\omega}{k}\right)^2. \quad (21)$$

This is the basic equation linking relative capability,  $k = (u/c_A)/(v/c_B)$  to the relative wage rate  $\omega = w_A/w_B$ . In the limiting case where  $m$  is large, so that  $r/(m - r)$ , and so  $a$ , converges to zero, this assumes the very simple form

$$\omega - \frac{1}{\omega} = \eta\left(k - \frac{1}{k}\right). \quad (22)$$

This can be written as a quadratic, but it is more informative to illustrate the solution graphically, as shown in Figure 3.

Beginning from the symmetric solution ( $u = v, c_A = c_B$ ) described above,  $\omega$

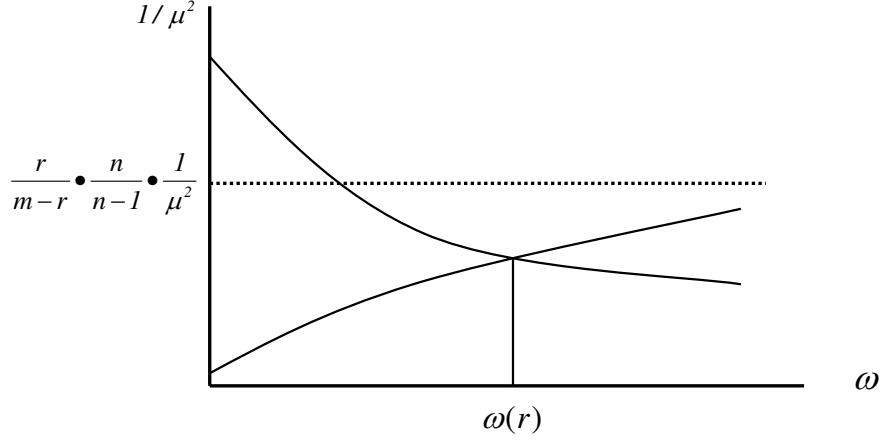


Figure 3: Relative Capability ( $k$ ) and relative wages ( $\omega$ ) in regime II. The case shown is where  $k > 1$ ,  $\omega > 1$  and  $\omega(k)$  is the equilibrium wage ratio. A rise in  $k$  shifts the falling schedule upwards. In the limiting case  $m \rightarrow \infty$  the falling schedule becomes flat, as shown by the hatched line.

risers monotonically with  $k$  through zone II. Accompanying this rise in  $\omega$  is a monotonic fall<sup>9</sup> in  $x_v$ , as can be checked directly from equation (12).

Since each of the goods  $r + 1, \dots, m$  is produced only by its  $n$  producers in country A, we have

$$p_u = \frac{n}{n-1} (w_A + w_C \mu) c_A \quad (23)$$

$$x_u = \frac{S}{m} \frac{n_1 - 1}{n_1^2} \frac{1}{(w_A + w_C \mu) c_A} \quad (24)$$

<sup>9</sup>This relies on our assumption that  $n \geq 2$ , ie there are at least 2 producers of each good in each country. If  $n = 1$ , then  $x_v$  remains constant as  $k$  varies, i.e. viability is maintained. Here, we never reach zone III.

which replace equations ((9) (12)) above. Products 1 to  $r$  are still produced in country B only whence their prices and outputs equal

$$p_1 = \frac{n_1}{n_1 - 1} (w_B + w_C \mu) c_A \quad (25)$$

$$x_{1B} = \frac{S}{m} \frac{n_1 - 1}{n_1^2} \frac{1}{(w_B + w_C \mu) c_1} \quad (26)$$

as in (17) above.

The labour supply equations, as before, imply that

$$\frac{L_S^A}{L_S^B} = \frac{w_A}{w_B} = \frac{L_D^A}{L_D^B} = \frac{(m - r) c_A n x_u}{r c_1 n_1 x_{1B}} \quad (27)$$

which on substituting for  $x_u, x_1$  from (24), (26) implies, in the limit  $n_1 \rightarrow \infty$ ,

$$\frac{w_A}{w_B} = \frac{m - r}{r} \frac{c_A}{c_1} \frac{n - 1}{n} \frac{(w_B + w_C \mu) c_1}{(w_A + w_C \mu) c_A} \quad (28)$$

As before, we have  $w_C = \mu(w_A + w_B)$ . Substituting this yields, on simplifying, and again writing  $w_A/w_B$  as  $\omega$ ,

$$\frac{\frac{r}{m-r} \frac{n}{n-1}}{(1 + \mu^2) \frac{1}{\omega} + \mu^2} = \frac{1}{(1 + \mu^2) \omega + \mu^2} \quad (29)$$

This equation is illustrated in Figure 4. As  $m$  increases, the rising schedule corresponding to the l.h.s. of (29) falls towards the horizontal axis, and

$w_A/w_B \rightarrow \infty$ . This will be relevant in what follows, where we need to specify the relative wage in zone III and so on the II/III boundary, to which we now turn.

One final remark is in order: in the limiting case  $\mu = 0$ , (29) reduces to

$$\frac{r}{m-r} \frac{n}{n-1} \omega = \frac{1}{\omega} \text{ whence } \frac{w_B}{w_A} = \frac{1}{\omega} = \sqrt{\frac{r}{m-r} \frac{n}{n-1}}.$$

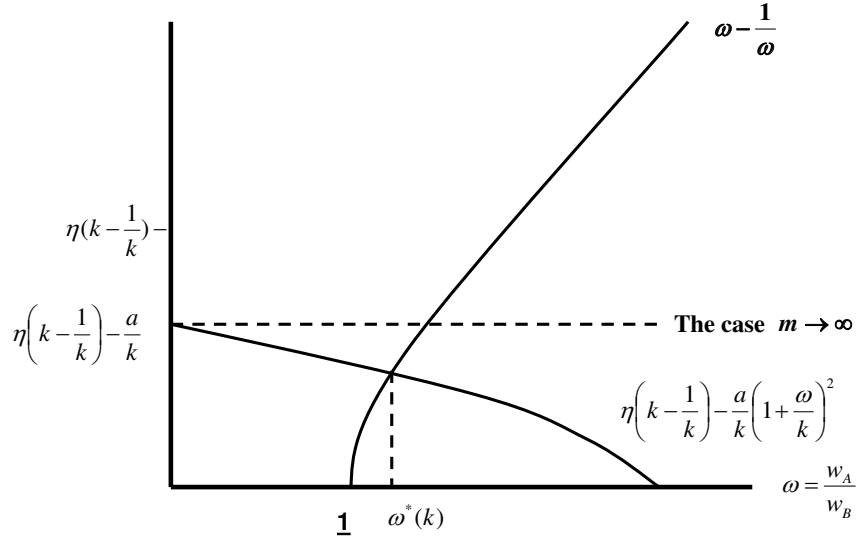


Figure 4: The equilibrium wage ratio  $w = w_A/w_B$  in zone III. Note that the falling schedule is independent of  $m$ ; as  $m$  rises, the rising schedule shifts downwards and  $\omega = w_A/w_B$  rises.

We now turn to the critical values of  $v$  and  $c_B$  which, for given values of  $u$  and  $c_A$ , correspond to the boundary between zones II and III at which country B's production of goods  $r + 1, \dots, m$  becomes non-viable. This boundary is

defined by the condition  $vx_v = 0$ , which using equation (12), implies

$$\frac{(w_A c_A + w_C \mu)}{(w_B c_B + w_C \mu)} \cdot \frac{v}{u} = \frac{n-1}{n}.$$

Writing  $w_C = \mu(w_A + w_B)$  as before, this becomes

$$\frac{w_A c_A + (w_A + w_B) \mu^2}{w_B c_B + (w_A + w_B) \mu^2} \cdot \frac{v}{u} = \frac{n-1}{n} \quad (30)$$

It is important to note the asymmetry between the quality parameters  $(u, v)$  and the productivity parameters  $(c_A, c_B)$  in expression (30); and to note that this asymmetry disappears in the special case  $\mu = 0$ , when (30) can be expressed as a condition on the capability ratio

$$\frac{k_A}{k_B} = \frac{u}{v} \cdot \frac{c_B}{c_A} = \frac{n}{n-1} \frac{w_B}{w_A} = \frac{n}{n-1} \frac{1}{\omega} \quad (31)$$

To explore the boundary condition (30), therefore, it is appropriate to proceed in two steps: first, we fix  $u = v$  and explore the condition on the  $c_B$ , for a given value of  $c_A$ . Second, we hold  $c_A = c_B$  and explore the condition on  $v$ , for a given value of  $u$ .

First, then, let  $u = v$ . Here, (30) leads, after some rearrangement, to

$$\left. \frac{c_B}{c_A} \right|_{crit} = \frac{n}{n-1} \cdot \frac{w_A}{w_B} \cdot \frac{1 + \left( \frac{w_B \mu^2}{w_A c_A} \right)}{1 + \left( \frac{w_B \mu^2}{w_A c_B} \right)} \quad (32)$$

In the limit  $m \rightarrow \infty$ , where the wage rate on the II/III boundary  $w_A/w_B \rightarrow$

$\infty$ , it follows that the critical ratio of  $c_B/c_A \rightarrow \infty$ . Here, any shortcoming in productivity in country B can be effectively offset by a sufficient fall in B's relative wage rate. This is the first key property of the solution.

We now turn to the quality threshold. Here, we set  $c_A = c_B = c$ ; (32) now leads after some re-arrangement to

$$\frac{v}{u} \Big|_{crit} = \frac{n-1}{n} \frac{\frac{w_B}{w_A} + \left(1 + \frac{w_B}{w_A}\right) \frac{\mu^2}{c}}{1 + \left(1 + \frac{w_B}{w_A}\right) \frac{\mu^2}{c}} \quad (33)$$

In the limit  $m \rightarrow \infty$ , where the wage rate on the II/III boundary  $w_A/w_B \rightarrow \infty$ , we have

$$\lim_{m \rightarrow \infty} \frac{v}{u} \Big|_{crit} = \frac{n-1}{n} \frac{1}{1 + c/\mu^2}. \quad (34)$$

Here, no fall in B's relative wage rate can offset a quality deficiency that exceeds the magnitude specified by (4); this is the analogue, in the present multi-country setup, of the single country result shown in Section 2.

### Welfare

Recall, by analogy with equation (8), that our welfare indicator for country B under autarky is

$$U_B^{Aut} = \frac{N}{2m^2} \left( \frac{v}{c_B + 2\mu^2} \right)^{2(m-r)/m} \left( \frac{1}{c_B + 2\mu^2} \right)^{2r/m} \quad (35)$$

which falls to zero with  $v$ , and with the productivity measure  $1/c_B$ . We begin by examining the (impact) effect of trade on country B's welfare by looking at

regime III. Here, the general expression for the welfare indicator,

$$U_B^{III} = \frac{1}{2} \left( \frac{w_B}{m} \right)^2 \prod_{i=1}^m \left( \frac{u_i}{p_i} \right)^{2/m} \quad (36)$$

becomes

$$\frac{1}{2} \left( \frac{w_B}{m} \right)^2 \mathbb{N} \left( \frac{u}{w_A c_A + w_C \mu} \right)^{2(m-r)/m} \left( \frac{1}{w_B c_1 + w_C \mu} \right)^{2r/m} \quad (37)$$

which on writing  $w_C = (w_A + w_B)\mu$  becomes

$$\frac{\mathbb{N}}{2m^2} \left( \frac{u}{\frac{w_A}{w_B} c_A + \left( \frac{w_A}{w_B} + 1 \right) \mu^2} \right)^{2(m-r)/m} \left( \frac{1}{c_1 + \left( \frac{w_A}{w_B} + 1 \right) \mu^2} \right)^{2r/m} \quad (38)$$

whence

$$\frac{U_B^{III}}{U_B^{Aut}} = \frac{u}{v} \frac{c_A}{c_B} \left( \frac{1 + 2\mu^2/c_B}{\frac{w_A}{w_B} c_A + \left( 1 + \frac{w_A}{w_B} \right) \mu^2} \right)^{2(m-r)/m} \left( \frac{1 + 2\mu^2/c_1}{1 + \left( 1 + \frac{w_A}{w_B} \right) \frac{\mu^2}{c_1}} \right) \quad (39)$$

Write  $w_A/w_B$  as  $\omega$ , and solve for the capability ratio that leads to  $U_B^{III} = U_B^{Aut}$ , viz

$$\frac{u}{v} \frac{c_A}{c_B} = \left( \frac{1 + 2\mu^2/c_B}{w + (1 + w)\mu^2/c_A} \right) \left( \frac{1 + 2\mu^2/c_1}{1 + (1 + \omega)\mu^2/c_1} \right)^{r/(m-r)} \quad (40)$$

This, then provides a criterion for determining whether the impact effect of trade on economy B raises or lowers our welfare indicator. The intuition is this:

opening up trade allows B's residents to access the superior products of quality  $u$  offered by A; but depresses their wage rates by cutting the demand for the goods they produce. Thus the welfare result turns on the interplay of two quantities: the quality ratio  $v/u$  and the wage ratio  $w_B/w_A$  in regime III. We focus on the boundary of zones II and III, where the wage rate is that characterizing zone III, and where the quality ratio  $v/u$  places B's products on the margin of viability. (When  $v/u$  falls below this level, the effect is to tilt the balance in favour of a welfare increase; for B's wage position does not deteriorate further, but the gain from accessing  $u$ , rather than  $v$ , increases. Hence if the impact effect of trade is negative for any  $v/u$ , it is so on the boundary of regimes II and III, as we will see in what follows).

To complete the analysis, we examine outcomes in  $(v/u, w_B/w_A)$  space, as shown in Figures 5 and 6. We examine the 'quality' case,  $u \neq v$ ,  $c_A = c_B = c$ .

In Figure 5, we show the critical value of  $v/u$  that characterizes the boundary of regimes II and III, as given by equation (33). As noted in the previous section, this schedule falls to a positive limiting value,  $[(n-1)/n]/(1+c/\mu^2)$  as  $v/u \rightarrow 0$ . We also show the criterion function we have just derived, equation (40): setting  $c_A = c_B = c$ , this expression gives the value of  $v/u$  which suffices to make B's residents indifferent between trade and autarky, for a given level of  $w_B/w_A$ . This schedule, as given by equation (40), rises from the origin as shown in Figure 5<sup>10</sup>.

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<sup>10</sup>For  $r \ll m$ , equation (40) takes a simple limiting form, which for  $c_A = c_B = c$  becomes  $\frac{v}{u} = \frac{1+2\mu^2/c}{\omega+(1+\omega)\mu^2/c}$ , whence as  $\mu \rightarrow \infty$ ,  $\frac{v}{u} \rightarrow \frac{1}{\omega}$ .



Three cases arise:

1. the regime III relative wage is relatively high and trade always raises  $U_B$ .

This will hold when  $r$  is large.

2. the regime III relative wage is low. Here two sub-cases arise, according as

$v(u)$  is (i) low, i.e. far below the level required for viability, in which case

$U_B$  again rises, or (ii) closer to the level required for viability, in which

case  $U_B$  falls as we move from autarky to trade.

These three cases, 1, 2(i) and 2(ii), are illustrated in Figures 5 and 6.

## 4 The Globalization Process

### Summary (Section in Preparation)

The short-run ‘impact’ effect of trade liberalization explored in the preceding section constitutes the first of the three phases of the globalization process. The second phase rests on the liberalization of investment flows, and is driven by the incentives firms face to create or transfer to low wage endowments. The channels through which this is effected are familiar, running from outsourcing to foreign direct investment. The relative importance of these channels varies widely from one industrial sector to another, as does the speed and effectiveness of transfer. (See, for example, the contrasting experience of industry, as against the machine-tool industry, reported in Sutton (2000,2003)). In what follows, we do not attempt to model the process of transfer, but focus on its effects. We

introduce the parameter  $t$  ( $\leq 1$ ) to represent the effectiveness of this transfer process, supposing that in Phase II, country B advances from quality level  $v$  to quality level  $tu$ , and similarly for productivity. So long as  $tu$  lies above the crossing point on Figure 6, country B's welfare level, at the end of Phase II, exceeds its original (autarky) level.

Phase III of the process involves the adjustment of firms' investment levels of capability building. We take the levels of quality  $u$  and  $tu$  emerging at the end of Phase II as initial conditions, and likewise for productivity. Each firm  $i$  now chooses a new quality level  $u^*$ , and pays a fixed and sunk cost  $F(u'_i) - F(u_i)$ , where  $u_i$  is its initial level, and similarly for productivity. We seek a Nash equilibria in  $(u, 1/c)$ , as in Section 2.

The outcome depends crucially on the 'initial conditions' emerging from the transfer process of Phase II. We distinguish two limiting cases, corresponding 'zero transfers' and 'full transfers' respectively. In the 'zero transfer' case, we assume that country B's capability remains at its initial level. Here, if the initial capability gap is sufficiently wide, the only Nash equilibrium outcome is one in which firm in country A advance their capabilities to reflect the increased size of the global market, while those in country B do not invest; depending on their initial level of capability, they may or may not remain active.

In the 'full transfer' limit, we begin from initial conditions in which all  $2n$  producers of each good  $r+1, \dots, m$  have the same level of capability. Here, the Nash equilibrium outcome is not unique. In general, there will be a set of Nash equilibria, which involve the survival of  $N \leq 2n$  'high capability' producers,

with a ‘fringe’ of  $2n - N$  firms who do not make any (incremental) investment in capability in Phase III. These ‘fringe’ firms may or may not remain active (i.e. have positive sales revenue at equilibrium). The equilibrium level of capability achieved by the high-level group varies inversely with the number of firms in this group: in equilibria with more firms, these firms have lower capability levels.

What can be said, in general, regarding the set of final equilibria emerging from Phase III? The answer turns on the parameter  $\beta$  ( $\geq 1$ ) which measures the effectiveness of fixed and sunk outlays in capability building. Recall that a low value of  $\beta$  leads to a high level of market concentration, i.e. a small number of firms. In this setting, it turns out that shakeout necessarily occurs in Phase III: the number of firms making incremental investments in capability is strictly less than  $2n$ , and the remaining ‘fringe’ of  $2n - N$  non-investing firms are inactive (make zero sales) at equilibrium. When  $\beta$  is large, on the other hand, (so that market concentration is low, i.e. the initial number of firms is large), then shakeout may or may not occur. There will be a Nash equilibrium in which all  $2n$  firms make a (small) incremental investment, leading to a symmetric solution. The welfare impact of Phase III investments varies with the degree of shakeout; higher levels of shakeout are associated with higher welfare gains.

## 5 Conclusions

The analysis set out in this paper has a number of key features:

- (i) ‘Competitiveness’: It is a feature of the type of model considered here that

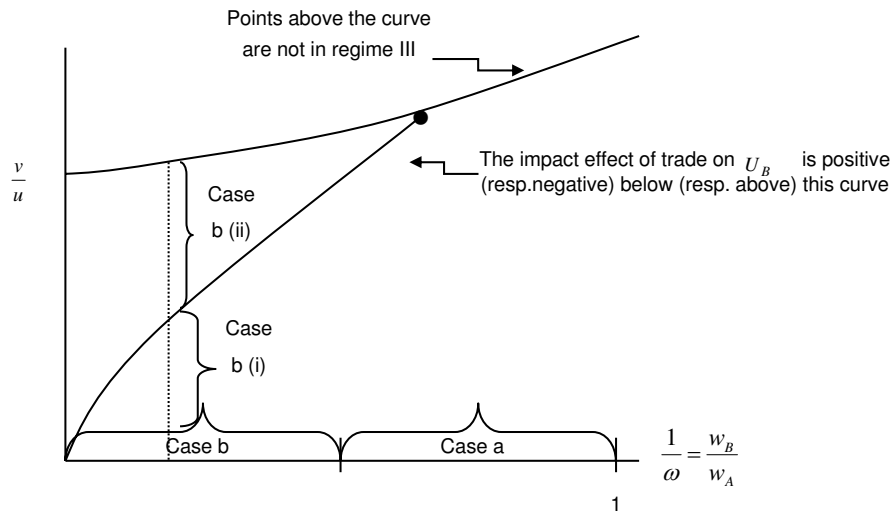


Figure 5:

relative quality matters, not only at the level of the firm, but also at the level of the country. A rise in other countries' quality levels can reduce domestic welfare; and international differences in relative wages cannot compensate for quality levels that fall below a certain threshold.

- (ii) The gains from globalization for counties with intermediate levels of initial capability derive primarily from the process of capability transfers (Phase II).
- (iii) The degree to which these transfers operate has a crucial influence on the pattern of firm survival, once firms adjust to the new 'global' environment (Phase III).

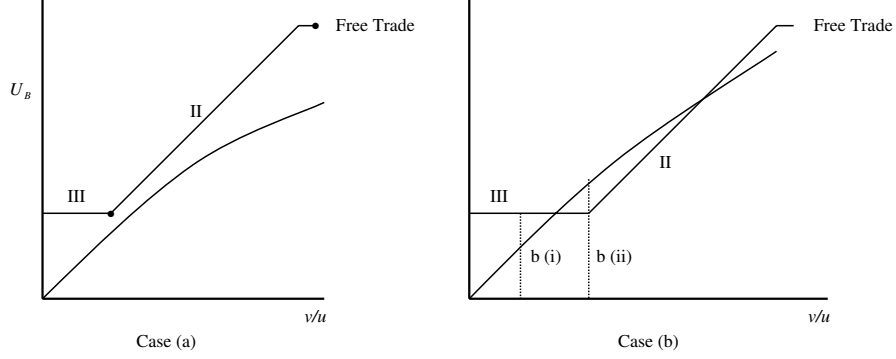


Figure 6:

## 6 Appendix

### Proof of Proposition 2

The proof is similar to the proof set out in Sutton (1991), Chapter 3, for the ‘quality’ case.

From the profit function (3) we obtain, on differentiating with respect to  $k_i$  and setting  $k_i = k_j = k$  (to characterize a symmetric Nash equilibrium), we obtain

$$\left. \frac{d\pi_i}{dk_i} \right|_{k_i=k} = -2 \frac{(n-1)^2}{n} \cdot \frac{S}{n^2} \cdot \frac{1}{k} \quad (\text{A1})$$

Recalling that  $k_i = (wc_i + \mu p_0) / u_i$  we have

$$\frac{dk_i}{du_i} = -\frac{k_i}{u_i} \quad ; \quad \frac{dk_i}{dc_i} = \frac{c_i}{1 + \frac{\mu p_0}{wc_i}} \quad (\text{A2})$$

From (A1) and (A2), and the cost function  $F(u, c) = u^\beta + c^{-\gamma}$ , we may write the first order conditions

$$\frac{dF}{du_i} = \frac{d\pi}{du_i} \quad \text{and} \quad \frac{dF}{dc_i} = \frac{d\pi}{dc_i}$$

at a symmetric equilibrium,  $u_j = u$  and  $c_j = c, \forall j$ , in explicit form, viz.

$$u^\beta = \frac{2(n-1)^2 S}{\beta n n^2} \tag{A3}$$

$$c^{-\gamma} = \frac{2(n-1)^2 S}{\gamma n n^2} \cdot \frac{1}{1 + \frac{\mu p_0}{wc}} \tag{A4}$$

Free entry implies a zero profit condition, which (ignoring integer effects) recalling that equilibrium profit per firm in a symmetric equilibrium equals  $S/n^2$  (from (3)), takes the form

$$u^\beta + c^{-\gamma} = S/n^2 \tag{A5}$$

Equations (A3), (A4) and (A5) characterize the equilibrium values of  $n$ ,  $u$  and  $c$  as a function of  $S$ .<sup>11</sup>

Note that the expression  $1/(1 + \frac{\mu p_0}{wc})$  must lie between 0 and 1 for all  $c$ . It follows from (A3) and (A4) that total fixed outlays per firm at equilibrium

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<sup>11</sup>Allowing for integer effects leads to the conclusion that the equilibrium number of firms is the integer part of  $n$ , as defined; and the equilibrium values of  $u$  and  $1/c$  are then defined by (A3, A4, A5), where  $n$  is replaced by its integer part.

satisfy

$$u^\beta + c^{-\gamma} = \frac{(n-1)^2}{n} \cdot \frac{S}{n^2} \cdot \frac{1}{2} \left\{ \frac{1}{\beta} + \frac{1}{\gamma} \frac{1}{1 + \frac{\mu p_0}{cw}} \right\} \quad (\text{A6})$$

Combining (A6) with the free entry condition (A3), and noting that  $0 \leq 1/(1 + \frac{\mu p_0}{cw}) \leq 1$  for all  $c \geq 0$  we obtain the bounds  $\underline{n}$  and  $\bar{n}$ , as specified in Proposition 2, as follows:

Note that for  $S \simeq 0$ , the free entry condition (A5) implies

$$u \simeq 0, \quad 1/c \simeq 0,$$

whence

$$\frac{1}{(1 + \frac{\mu p_0}{cw})} \simeq 1$$

and the first order conditions (A3), (A4) imply  $n \simeq \underline{n}$ . The case  $S \rightarrow \infty$  is less straightforward. Here, the first order condition (A3), together with the fact that  $n \leq \bar{n}$ , implies that  $u \rightarrow \infty$ . It is also the case that, as  $S \rightarrow \infty$ ,  $1/c \rightarrow 0$ ; to see this, suppose the contrary, viz. that there is some value  $c_0 > 0$  such that for any  $S_0$ , we can find some  $S > S_0$  at which  $c \geq c_0$ . Set  $c = c_0$  in (A4), letting  $S \rightarrow \infty$  and noting that  $\underline{n} \leq n \leq \bar{n}$ , we obtain a contradiction.

We can now show that as  $S \rightarrow \infty$ ,  $n \rightarrow \bar{n}$ . To see this, combine the free entry condition (A5) with the first order condition for  $u$ , (A3), to obtain

$$u^\beta = \frac{2}{\beta} \frac{(n-1)^2}{n} \frac{S}{n^2} = \frac{2}{\beta} \frac{(n-1)^2}{n} (u^\beta + c^{-\gamma}) \quad (\text{A7})$$

whence

$$\frac{2}{\beta} \frac{(n-1)^2}{n} = \frac{u^\beta}{u^\beta + c^{-\gamma}} \quad (\text{A8})$$

where the expression on the r.h.s. equals the share of fixed outlays spent on u ('product innovation'). We may compute this share by dividing the first order condition (A4) by (A3) to obtain

$$\frac{c^{-\gamma}}{u^\beta} = \frac{\beta}{\gamma} \left( 1 + \frac{\mu p_0}{wc} \right)$$

which implies that as  $S \rightarrow \infty$ , so that  $c \rightarrow 0$ , the share of spending on process innovation,  $c^{-\gamma} / (u^\beta + c^{-\gamma})$ , falls to zero, and so (A8) collapses to the defining equation for  $\bar{n}$ .

## 7 References

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